

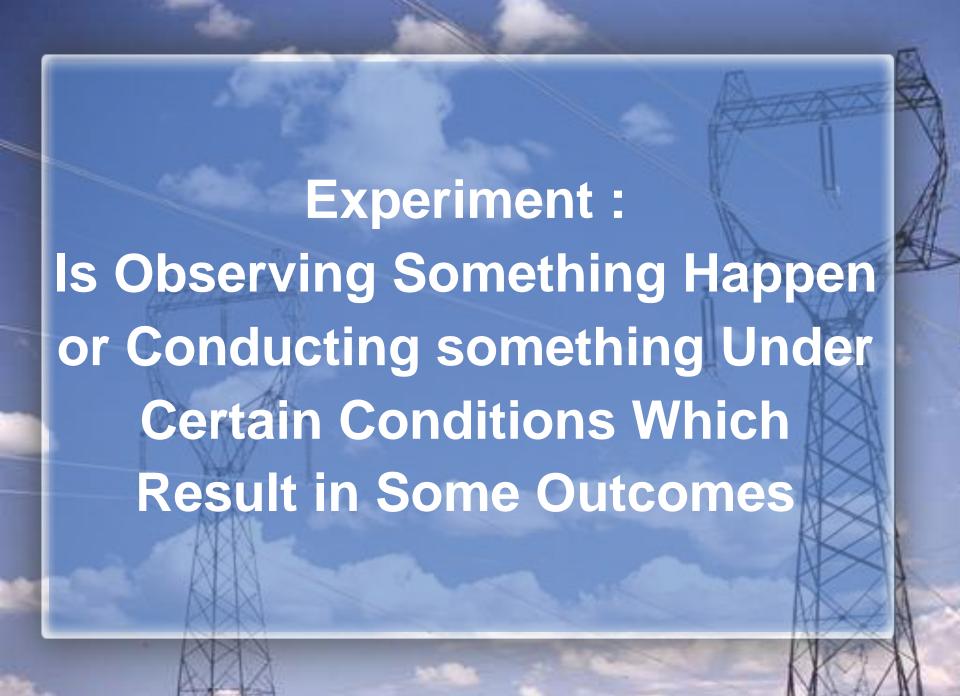






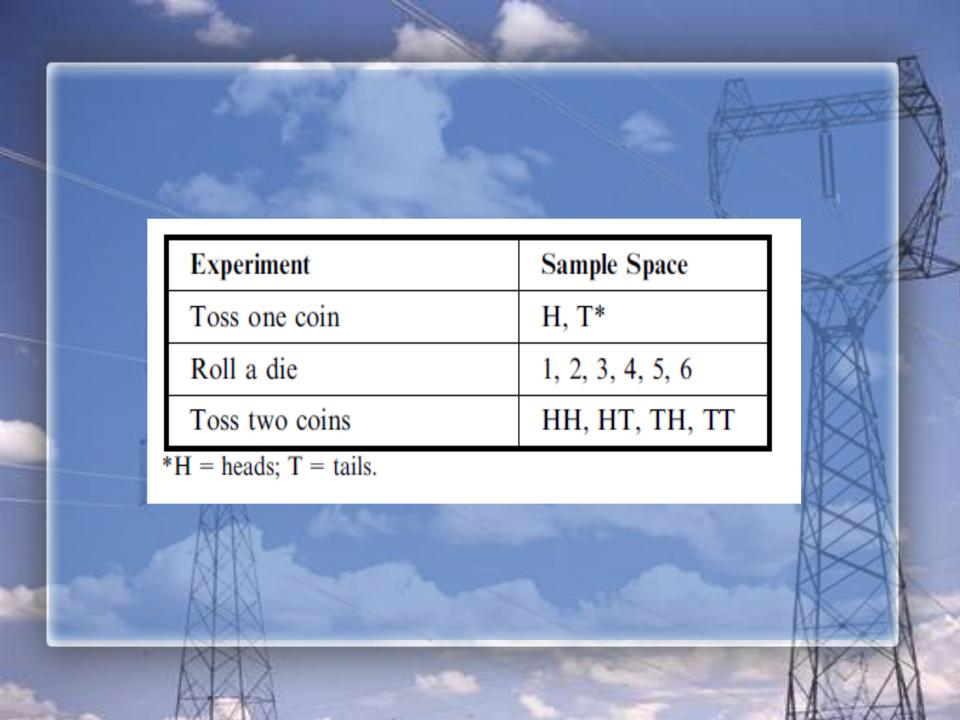
ریشه آن به بازی و شانس باز می گردد. قرن شانزدهم ورود دانشمنداني چون برنولي توسعه پاسکال، فرمت و فون میسس

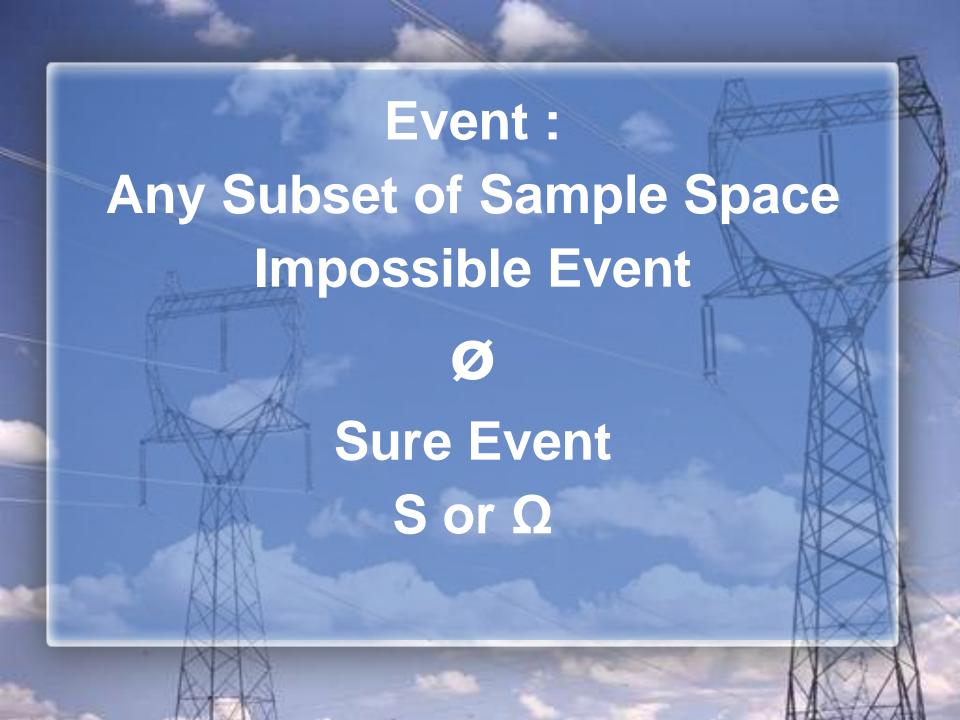




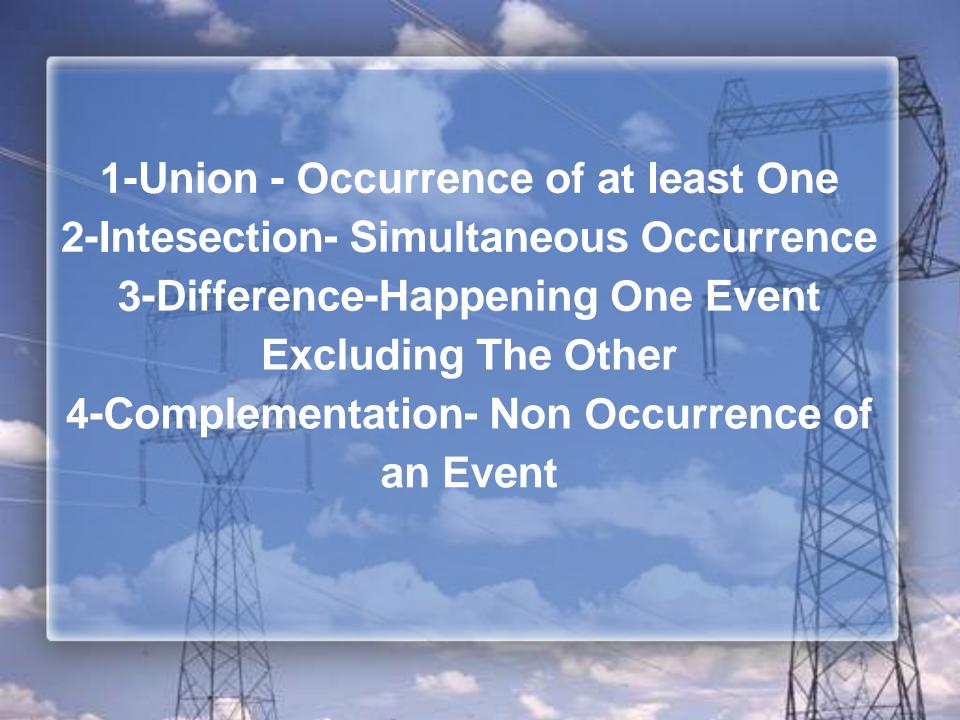














Addition Rule I: When two events are mutually exclusive,

$$P(A \text{ or } B) = P(A) + P(B)$$

**EXAMPLE:** When a die is rolled, find the probability of getting a 2 or a 3.

### **SOLUTION:**

As shown in Chapter 1, the problem can be solved by looking at the sample space, which is 1, 2, 3, 4, 5, 6. Since there are 2 favorable outcomes from 6 outcomes,  $P(2 \text{ or } 3) = \frac{2}{6} = \frac{1}{3}$ . Since the events are mutually exclusive, addition rule 1 also can be used:

$$P(2 \text{ or } 3) = P(2) + P(3) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$



**Addition Rule II:** If A and B are two events that are not mutually exclusive, then P(A or B) = P(A) + P(B) - P(A and B), where A and B means the number of outcomes that event A and event B have in common.

**EXAMPLE:** A die is rolled. Find the probability of getting an even number or a number less than 4.

#### **SOLUTION:**

Let A = an even number; then  $P(A) = \frac{3}{6}$  since there are 3 even numbers—2, 4, and 6. Let B = a number less than 4; then  $P(B) = \frac{3}{6}$  since there are 3 numbers less than 4—1, 2, and 3. Let (A and B) = even numbers less than 4 and  $P(A \text{ and } B) = \frac{1}{6}$  since there is one even number less than 4—namely 2. Hence,

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = \frac{3}{6} + \frac{3}{6} - \frac{1}{6} = \frac{5}{6}$$

The results of both these examples can be verified by using sample spaces and classical probability.



**Multiplication Rule I:** For two independent events A and B,  $P(A \text{ and } B) = P(A) \cdot P(B)$ .

**EXAMPLE:** A coin is tossed and a die is rolled. Find the probability of getting a tail on the coin and a 5 on the die.

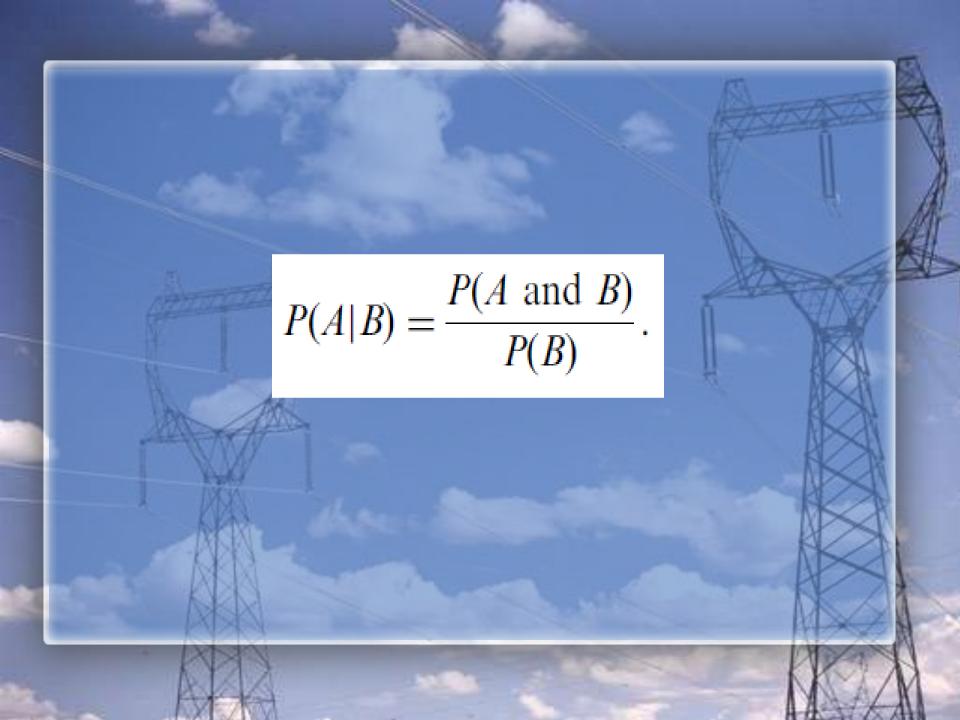
## **SOLUTION:**

Since  $P(\text{tail}) = \frac{1}{2}$  and  $P(5) = \frac{1}{6}$ ;  $P(\text{tail and } 5) = P(\text{tail}) \cdot P(5) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$ . Note that the events are independent.





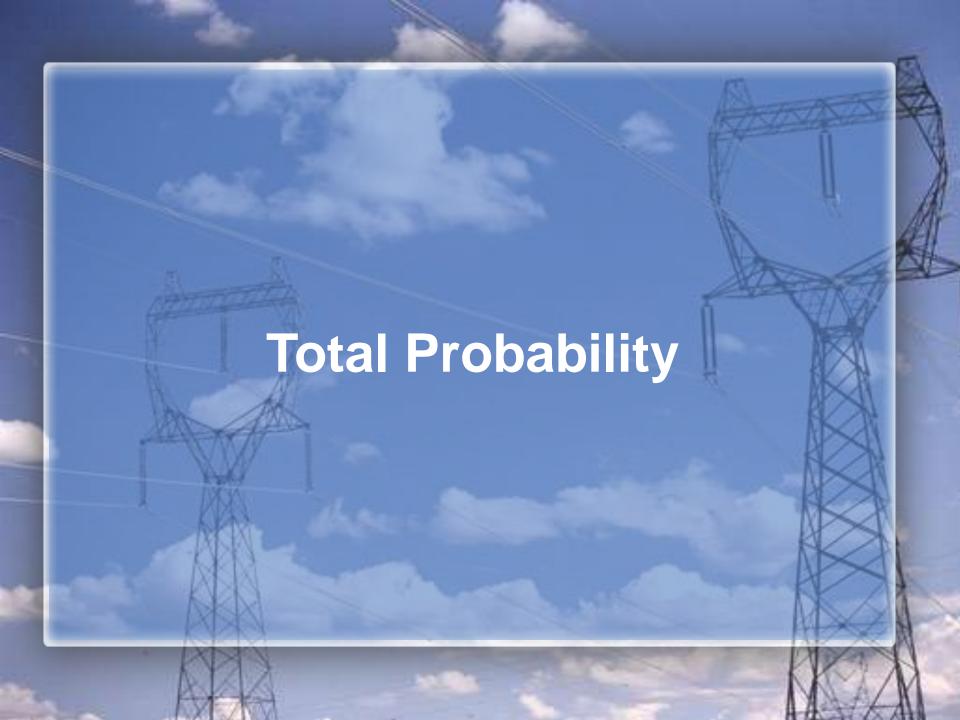




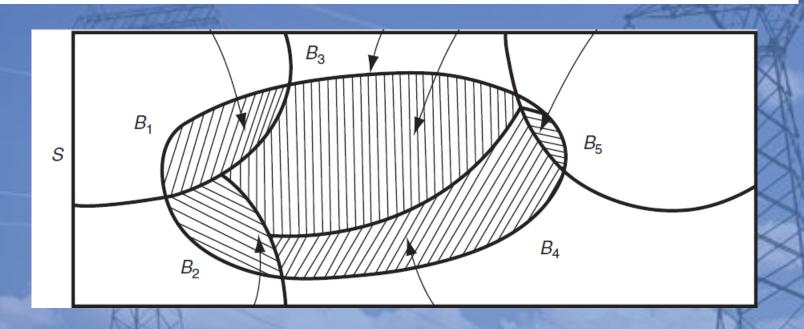
**EXAMPLE:** A die is rolled; find the probability of getting a 4, if it is known that an even number occurred when the die was rolled.

P(A and B) is the probability of getting a 4 and an even number at the same time. Notice that there is only one way to get a 4 and an even number—the outcome 4. Hence  $P(A \text{ and } B) = \frac{1}{6}$ . Also P(B) is the probability of getting an even number which is  $\frac{3}{6} = \frac{1}{2}$ . Now

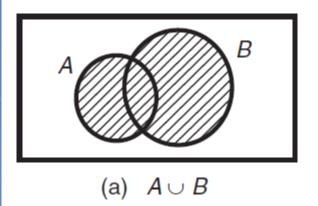
$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$
$$= \frac{\frac{1}{6}}{\frac{1}{2}}$$

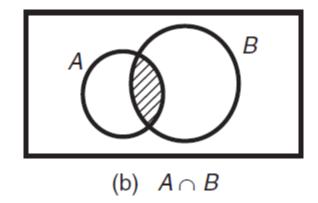


$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \dots + P(A|B_n)P(B_n)$$
  
=  $\sum_{j=1}^{n} P(A|B_j)P(B_j)$ .



# **Venn Diagrams**







# Axiomatic Probability Kolmogorov-1933

- Axiom 1:  $P(A) \ge 0$  (nonnegative).
- Axiom 2: P(S) = 1 (normed).
- Axiom 3: for a countable collection of mutually exclusive events  $A_1, A_2, \ldots$  in S,

$$P(A_1 \cup A_2 \cup \ldots) = P\left(\sum_j A_j\right) = \sum_j P(A_j)$$
 (additive).



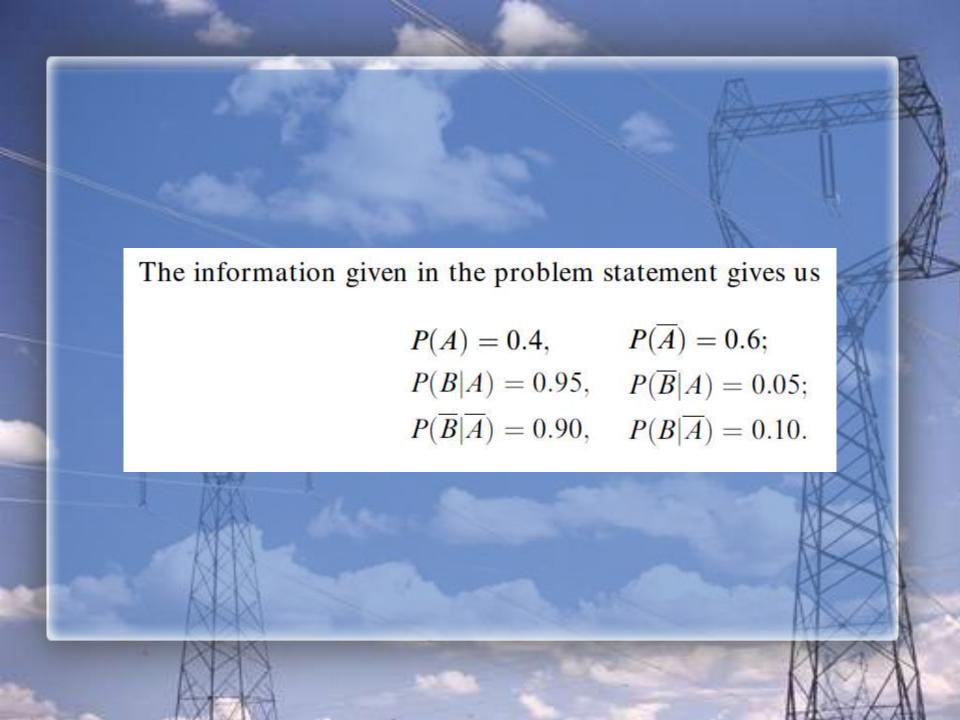
**Example 2.12.** Problem: a simple binary communication channel carries messages by using only two signals, say 0 and 1. We assume that, for a given binary channel, 40% of the time a 1 is transmitted; the probability that a transmitted 0 is correctly received is 0.90, and the probability that a transmitted 1 is correctly received is 0.95. Determine (a) the probability of a 1 being received, and (b) given a 1 is received, the probability that 1 was transmitted. Answer: let

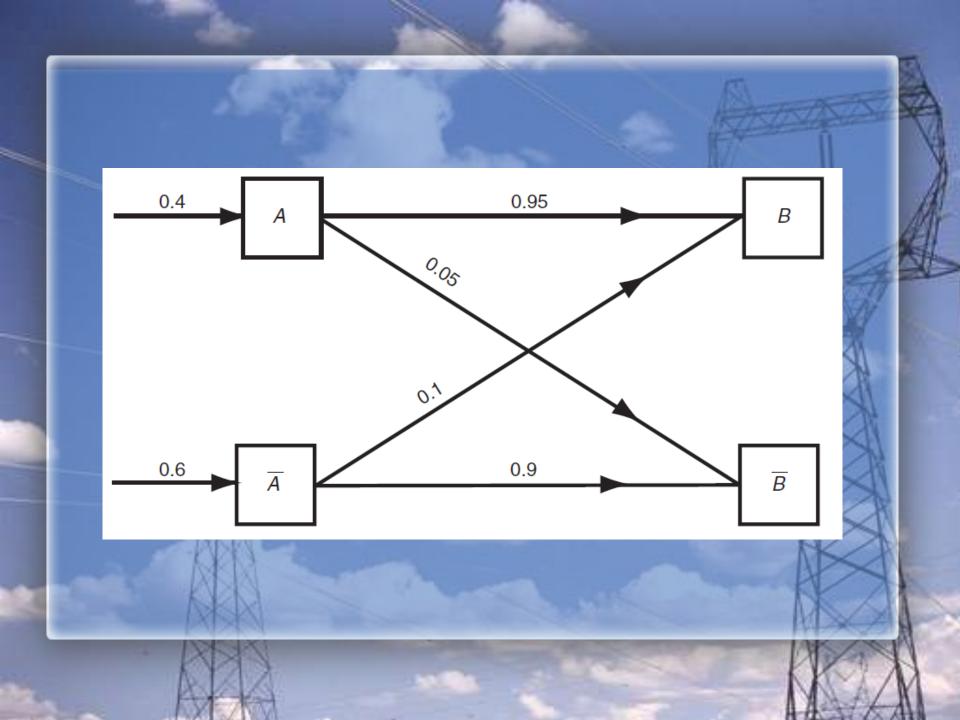
A =event that 1 is transmitted,

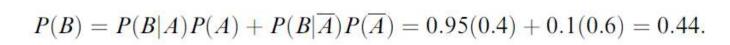
 $\overline{A}$  = event that 0 is transmitted,

B =event that 1 is received,

 $\overline{B}$  = event that 0 is received.

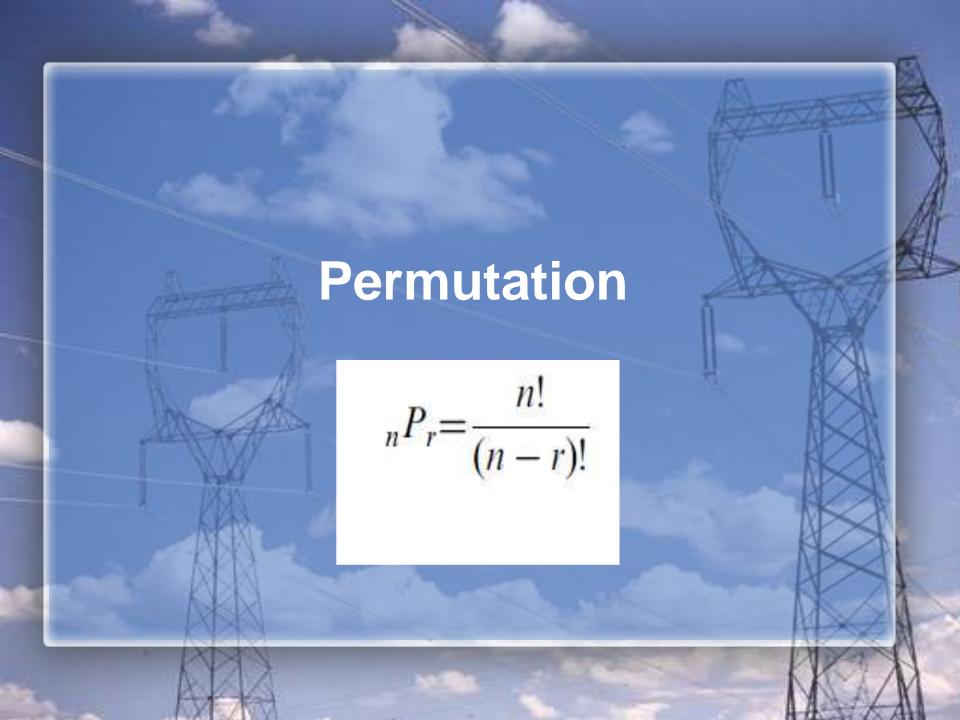


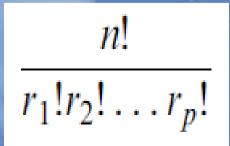




The probability of interest in part (b) is P(A|B), and this can be found using Bayes' theorem [Equation (2.28)]. It is given by:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{0.95(0.4)}{0.44} = 0.863.$$





**EXAMPLE:** How many different permutations can be made from the letters of the word **Mississippi**?

## **SOLUTION:**

There are 4s, 4i, 2p, and 1 m; hence, n = 11,  $r_1 = 4$ ,  $r_2 = 4$ ,  $r_3 = 2$ , and  $r_4 = 1$ 

$$\frac{11!}{4! \cdot 4! \cdot 2! \cdot 1!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{A}!}{\cancel{A}! \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1 \cdot 1} = \frac{1,663,200}{48} = 34,650$$

$$_{n}C_{r} = \frac{n!}{(n-r)!r!}$$

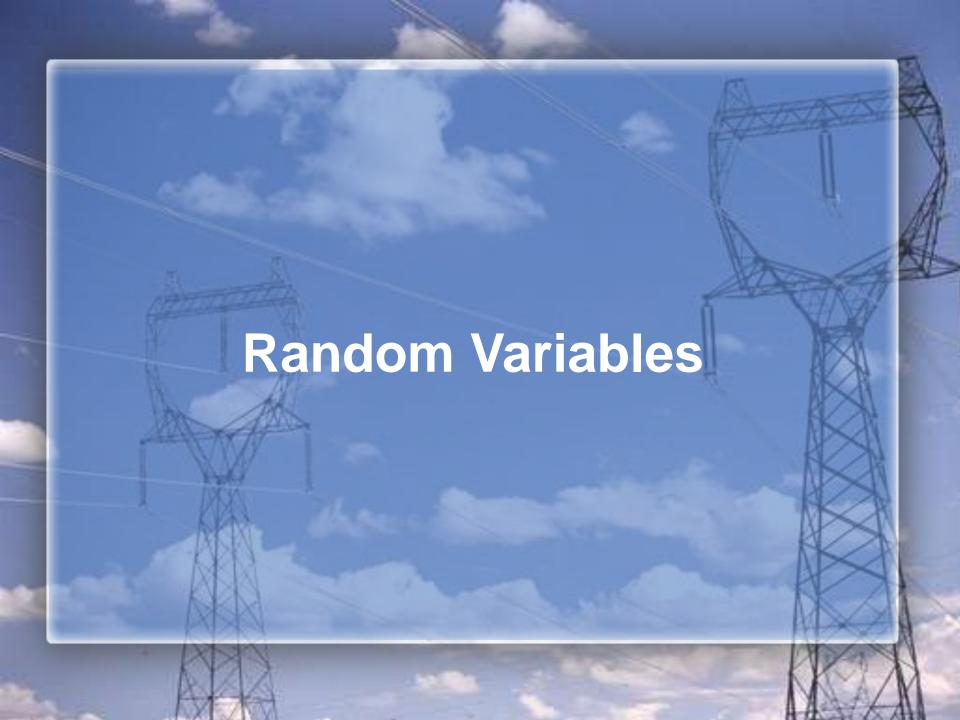
**EXAMPLE:** A salesperson has to visit 10 stores in a large city. She decides to visit 6 stores on the first day. In how many different ways can she select the 6 stores? The order is not important.

## **SOLUTION:**

Let n = 10 and r = 6; then

$$_{10}C_6 = \frac{10!}{(10-6)!6!} = \frac{10!}{4!6!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 6!} = 210$$

She can select the 6 stores in 210 ways.







مجموعه ای از اجزا که جهت دستیابی به هدفی واحد عمل می کنند. در یک شبکه توزیع، هدف واحد برقراری تغذیه الکتریکی مطمئن و با کیفیت است.

## مطالعه قابلیت اطمینان به روش جریان حالت: State Flow

تعریف حالت نرمال عملکرد سیستم:
همه کلیدها در وضعیت معمول خود هستند، هیچ تجهیز حفاظتی عمل نکرده است، همه المانها بصورت مناسبی عمل می کنند و بارگذاری همه عناصر در محدوده بارگذاری مجاز است

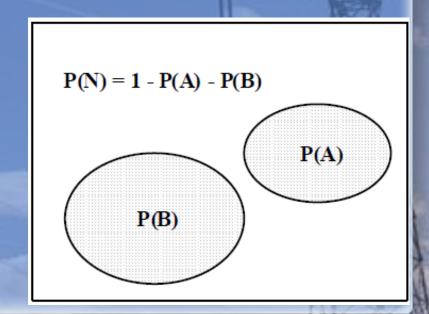
## مطالعه قابلیت اطمینان به روش جریان حالت: State Flow

تعریف Contingency:
هر تغییر احتمالی (غیر برنامه ریزی شده) در سیستم که باعث خارج آن از حالت کارکرد نرمالش بشود تعریف حادثه برنامه ریزی شده (Scheduled event):
هر فعالیت برنامه ریزی شده نظیر تعمیرات در سیستم که باعث خارج آن از حالت کارکرد نرمالش بشود

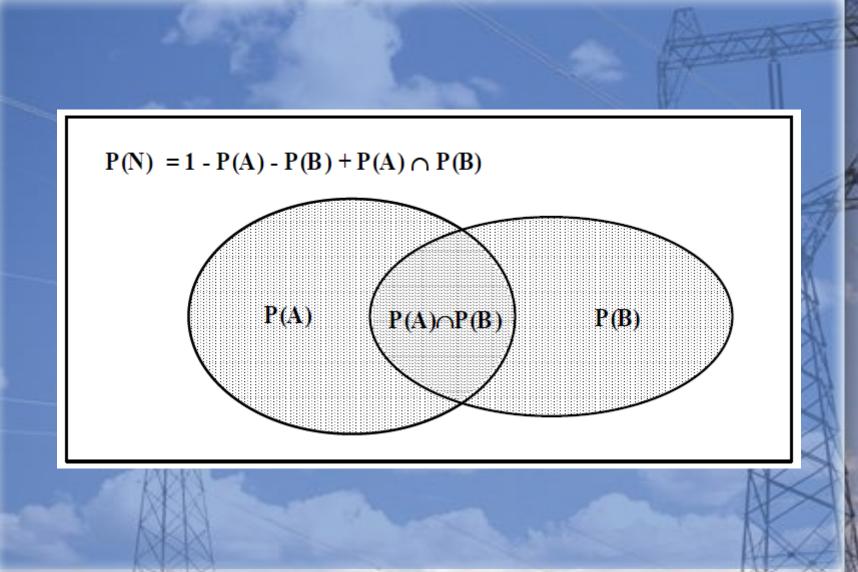
فضاى حالت (State Space):

فضایی شامل همه حالات ممکن است. مثلا اگر برای سیستم سه حالت نرمال، تعمیرات و خارج بودن از مدار متصور شود که از هم مستقل باشند، داریم:

P(A) + P(B) + P(C) = 1



حالات انحصاری (Mutually Exclusive): حالاتی هستند که رویداد همزمان آنها با هم غیر ممکن است. خیلی از سیستمها با Redundancy طراحی می شوند. در این حالت ممکن است هم Outage داشته باشیم و هم سیستم در حالت نرمال خود باشد. یعنی حالات سیستم مستقل نباشند



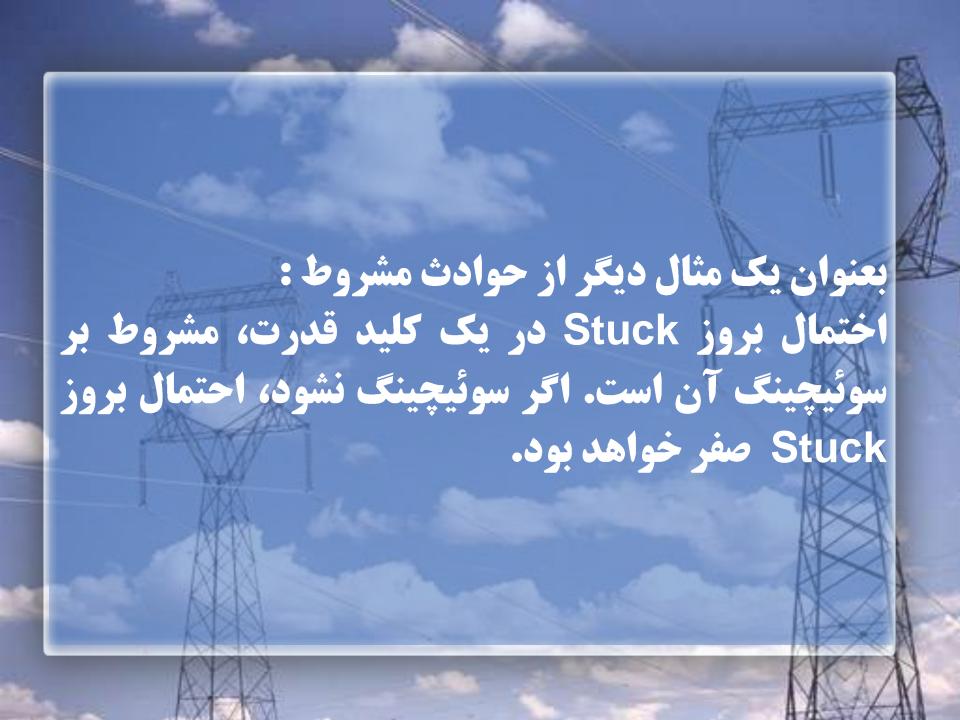
حوادث مستقل (Independent Events): حوادثی هستند که وقوع یکی بر دیگری تاثیر ندارد (مثل پرتاب دو تاس).

دو حادثه وابسته: شرایط آب و هوایی و بروز خطا

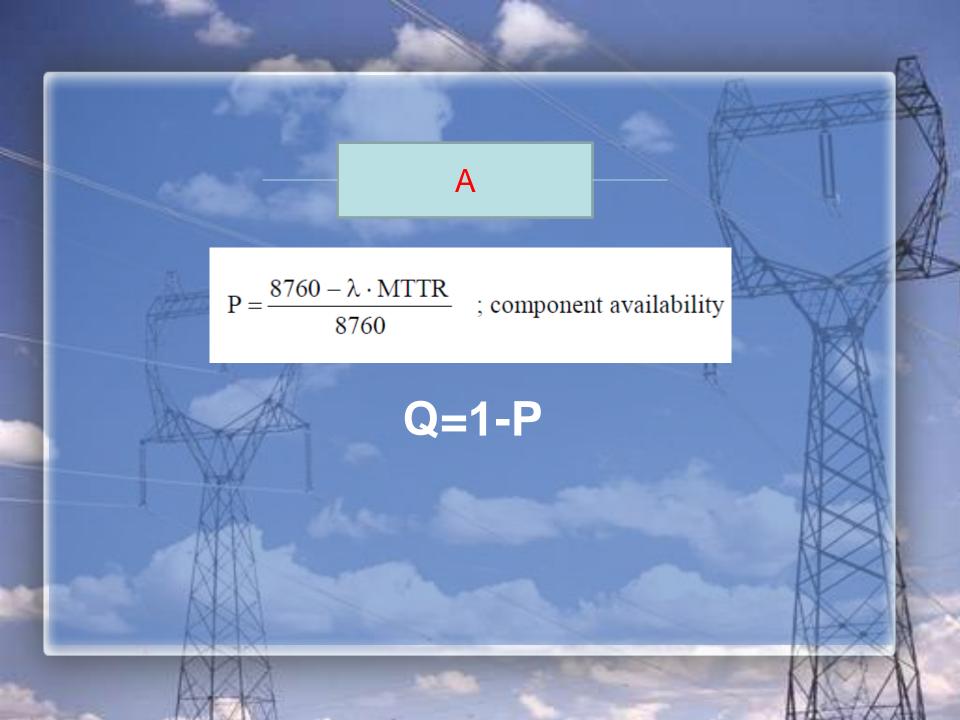
 $P(A \cap B) = P(A) \cdot P(B)$ ; true if A and B are independent

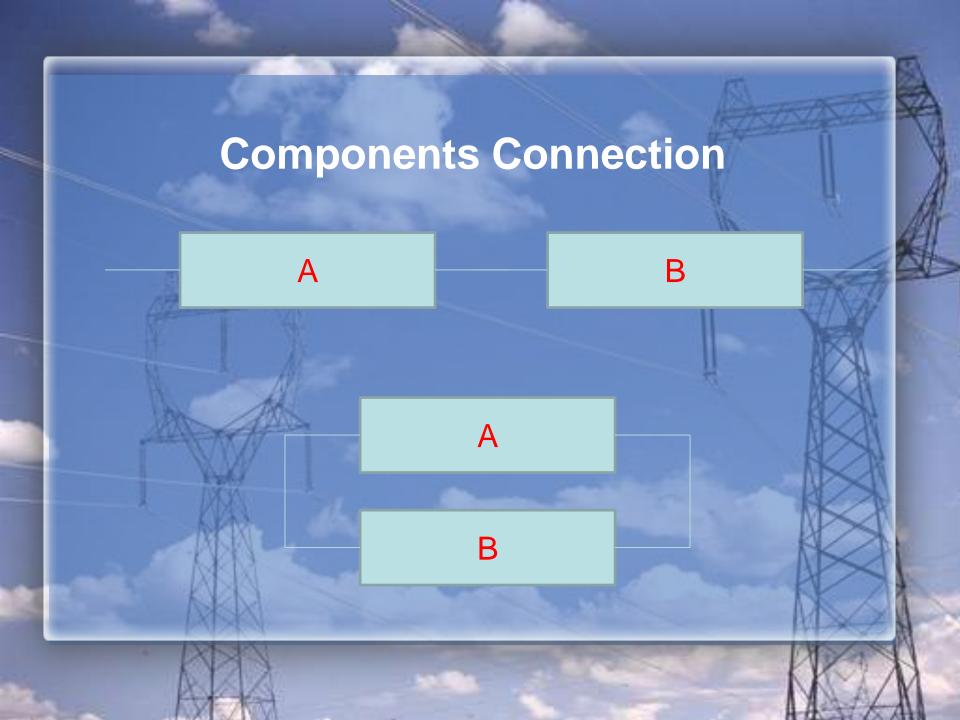
حوادث شرطی یا وابسته: احتمال وقوع یکی از روی احتمال وقوع یا عدم وقوع دیکری تعیین می شود

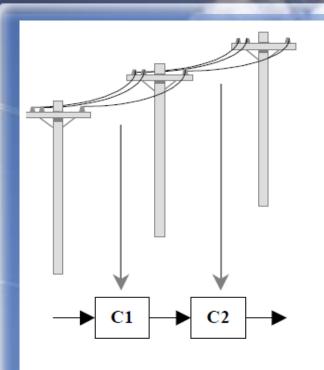
 $P(F_{operational} \mid B_{operating})$ 





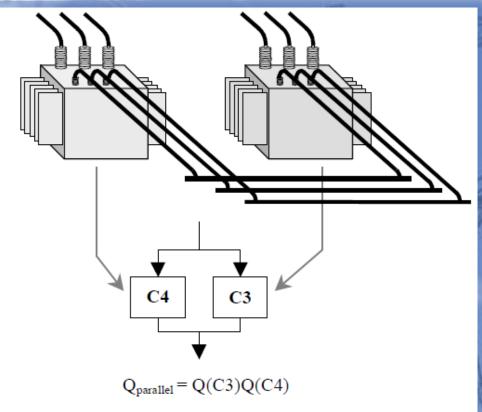






 $P_{\text{series}} = P(C1)P(C2)$ 

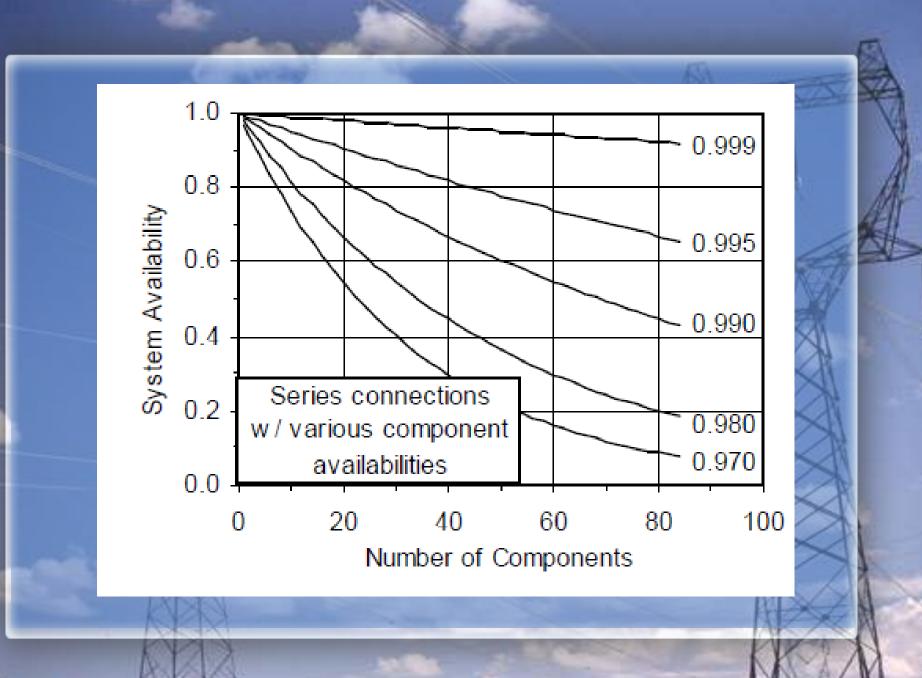
Series Connection

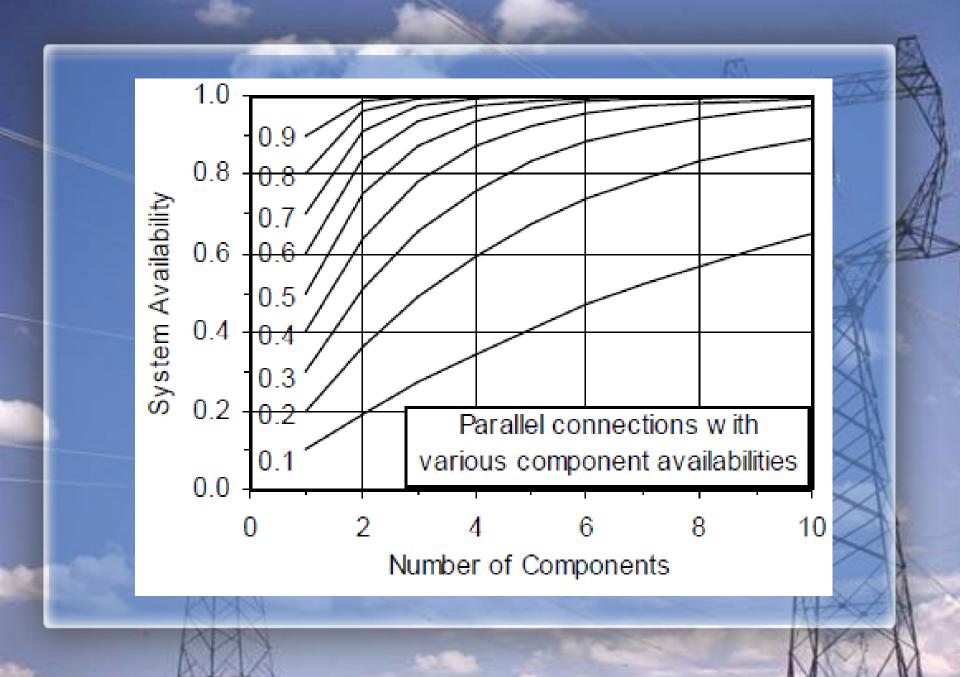


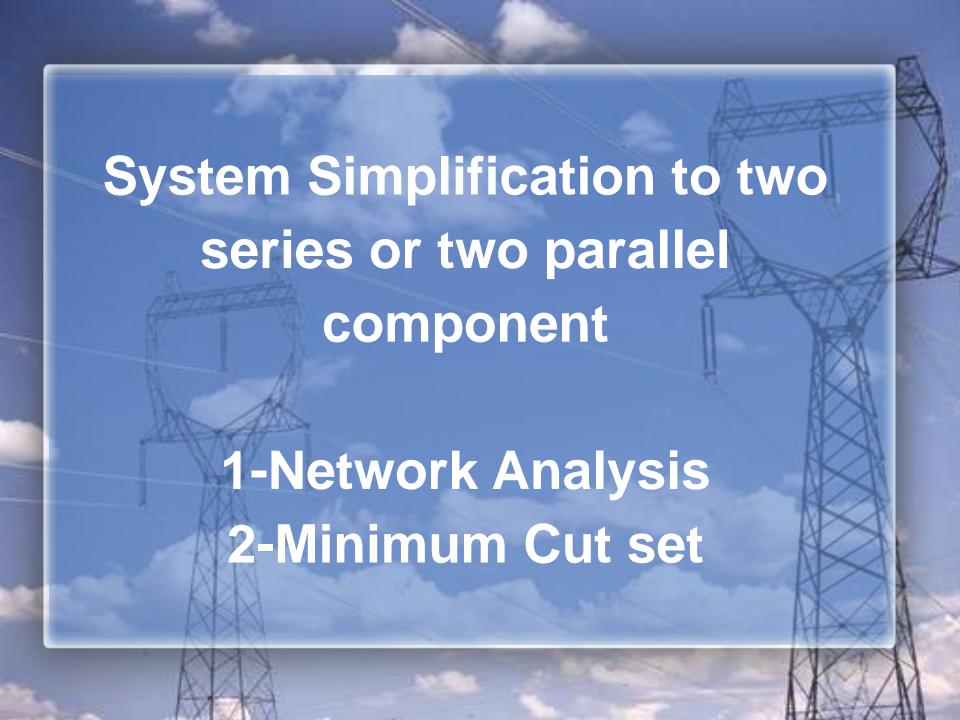
Parallel Connection

$$P_{\text{series}} = \prod P_{\text{component}}$$

$$Q_{\text{parallel}} = \prod Q_{\text{component}}$$





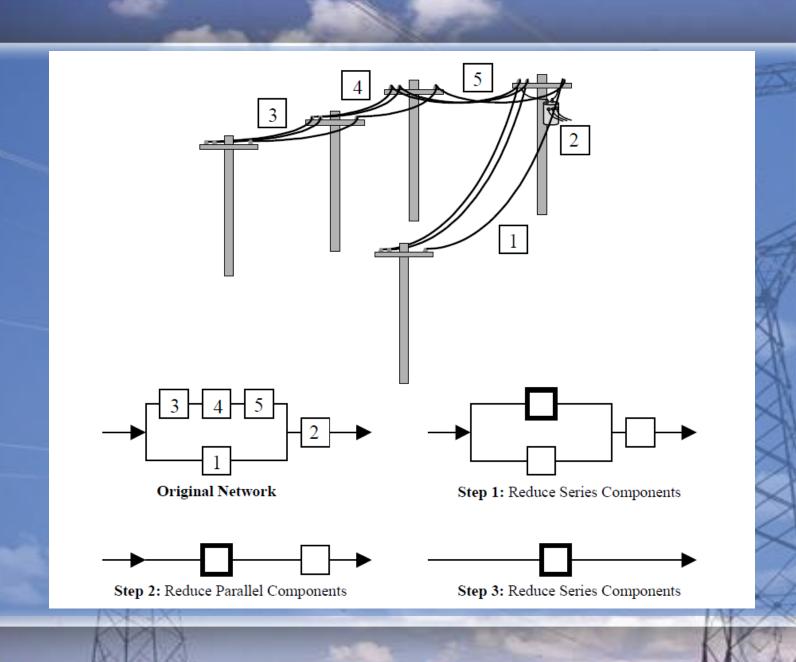


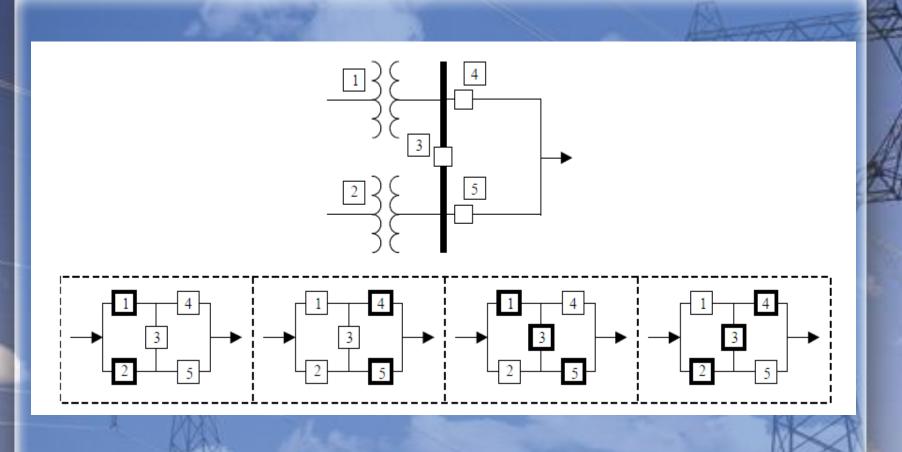
**Minimal Cut Set** — a set of n components that cause the system to be unavailable when all n components are unavailable but will not cause the system to be unavailable if less than n components are unavailable.

مزایای روش Cut Set: ۱- بر روی کامپیوتر دیجیتال قابل پیاده سازی است ۲- بهتر جواب می دهد

۳- دید مهندسی از عناصر بحرانی سیستم به دست می

دهد





Q=Q1+Q2+Q3+Q4



برای حل مساله قابلیت اطمینان در فضای حالت، ۳ روش مورد استفاده قرار می گیرد:

۱- مدلسازی مارکوف (MARKOV MODELLING) ۲- شبیه سازی تحلیلی (Analytical Simulation) ۳-شبیه سازی مونت کارلو (Monte Carlo Simulation)







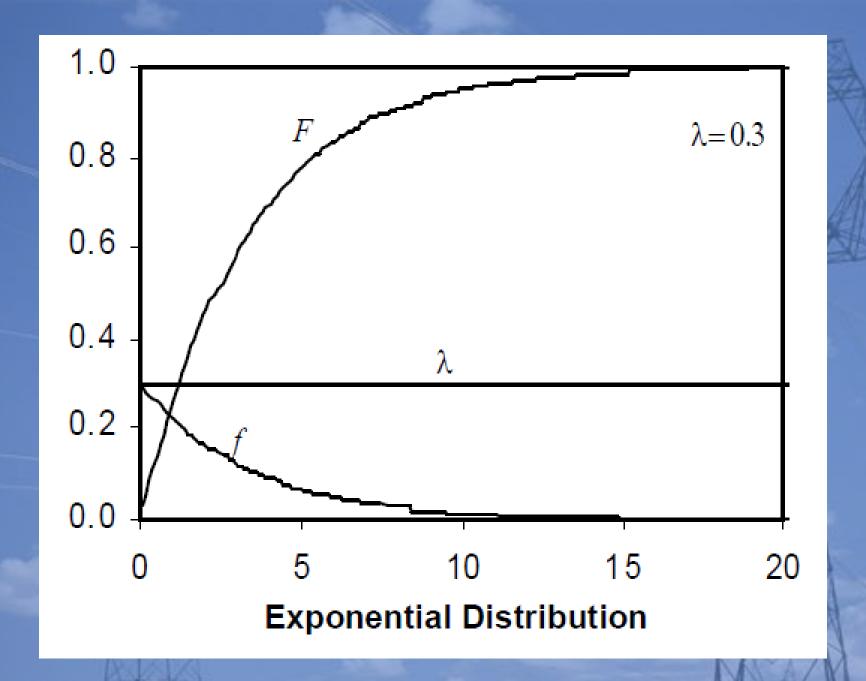


$$\lambda(x) = \frac{f(x)}{1 - F(x)}$$
;  $\lambda = \text{Hazard Function}$ 

$$R(x)=1-F(x)$$
;  $R = Survivor Function$ 

Expected Value = 
$$\overline{x} = \int_{-\infty}^{\infty} x \cdot f(x) dx$$

Variance = 
$$\int_{-\infty}^{\infty} [f(x) - \overline{x}]^2 dx$$



## 4.3.1 Normal Distribution

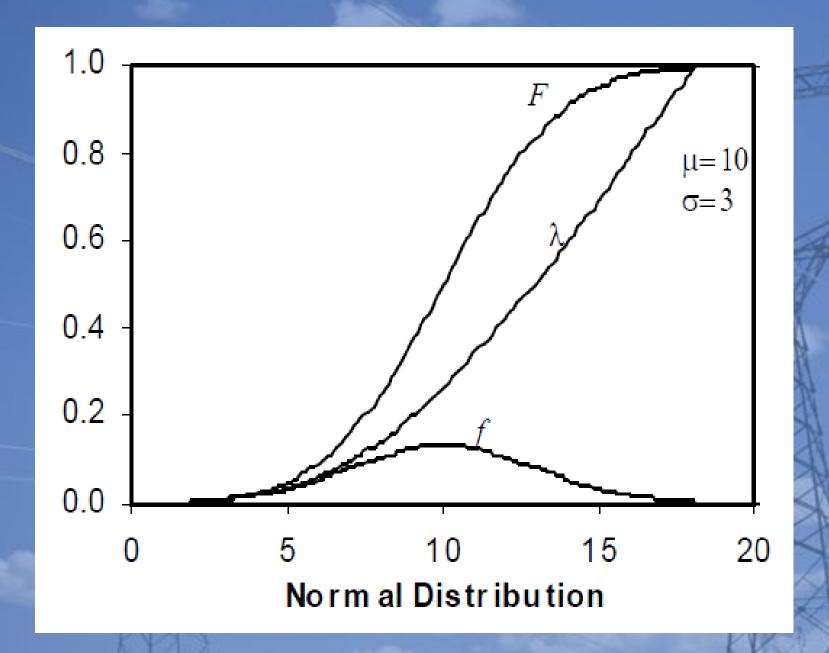
Perhaps the most well known distribution of all is the normal distribution—the proverbial "bell curve" that is mathematically characterized by its expected value and standard deviation. Formulae corresponding to the normal distribution are:

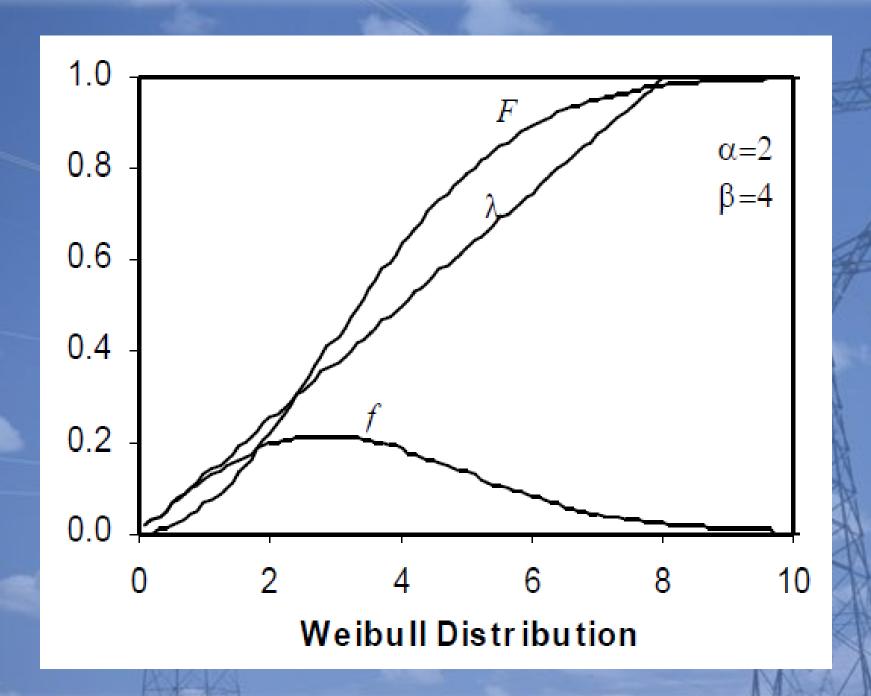
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]; -\infty \le x \le \infty$$
Expected Value =  $\mu$ 
Variance =  $\sigma^2$ 

$$(4.10)$$

$$\sigma = \sqrt{\text{Variance}}$$

Use of the normal distribution is so common that its parameters,  $\mu$  and  $\sigma$ , are often times misrepresented to be means and standard distributions for functions other than the normal distribution. This is not always true— $\mu$  and  $\sigma$  correspond to mean and standard deviation for the normal distribution, but not necessarily for other distributions.





$$f(x) = \frac{\beta x^{\beta - 1}}{\alpha^{\beta}} \exp \left| -\left(\frac{x}{\alpha}\right)^{\beta} \right| ; \quad x \ge 0$$

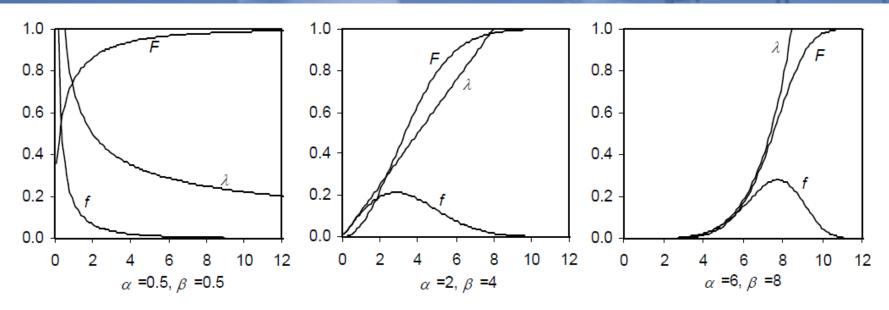
$$F(x) = 1 - \exp \left[ -\left(\frac{x}{\alpha}\right)^{\beta} \right]$$

$$\lambda(x) = \frac{\beta x^{\beta - 1}}{\alpha^{\beta}}$$

; Weibull Distribution

Expected Value = 
$$\alpha \Gamma \left( \frac{1}{\beta} + 1 \right)$$

Variance = 
$$\alpha^2 \left[ \Gamma \left( \frac{2}{\beta} + 1 \right) - \Gamma^2 \left( \frac{1}{\beta} + 1 \right) \right]$$



**Figure 4.6.** Thee different Weibull distributions. By varying the scale parameter,  $\alpha$ , and the shape parameter,  $\beta$ , a wide variety of distribution shapes can be modeled. The left graph represents an exponentially decaying density curve, the middle graph shows a density curve that is skewed to the left, and the right graph shows a density curve that is skewed to the right.

$$\beta = \text{Slope of Linear Fit}$$

$$\alpha = \exp\left(-\frac{y - \text{intercept}}{\beta}\right)$$

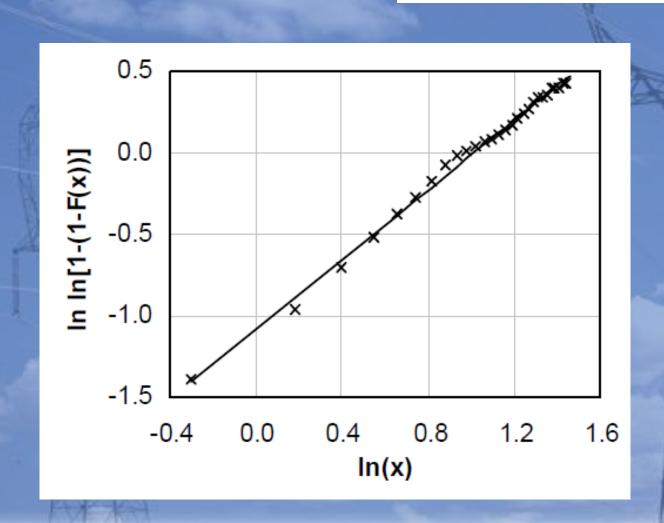
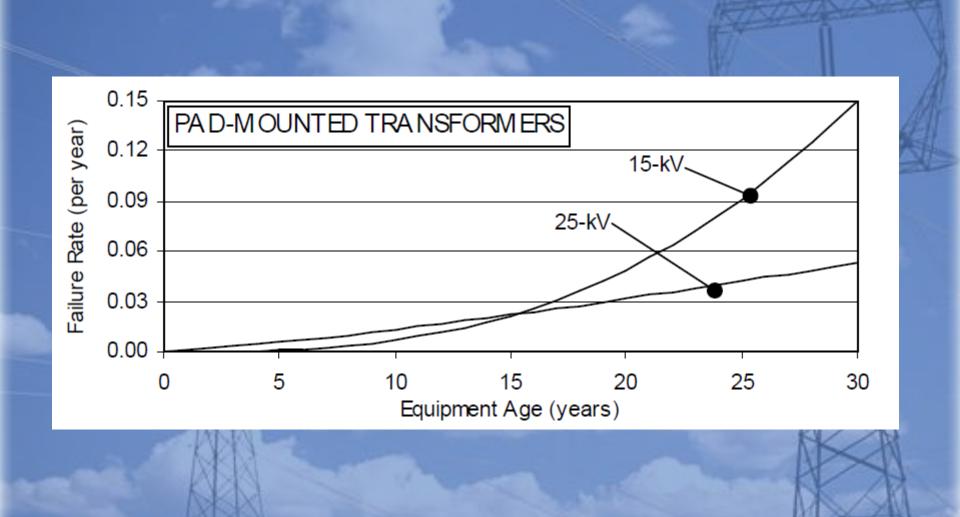


 Table 4.2 Reliability of underground distribution components.

Description	<b>λ</b> <sub>P</sub> (per year)			MTTR (hours)		
	Low	Typical	High	Low	Typical	High
Underground Cable						
Primary Cable	$0.003^{*}$	$0.070^*$	0. 587*	1.5	10.0	30
Secondary Cable	$0.005^*$	$0.100^*$	$0.150^*$	1.5	10.0	30
Elbows Connectors	6.0e-5	0.0006	0.001	1.0	4.5	8.0
Cable Splices and Joints	6.0e-5	0.030	0.159	0.5	2.5	8.0
Padmount Transformers	0.001	0.010	0.050	4.0	6.5	7.8
Padmount Switches	0.001	0.003	0.005	0.8	2.5	5.0

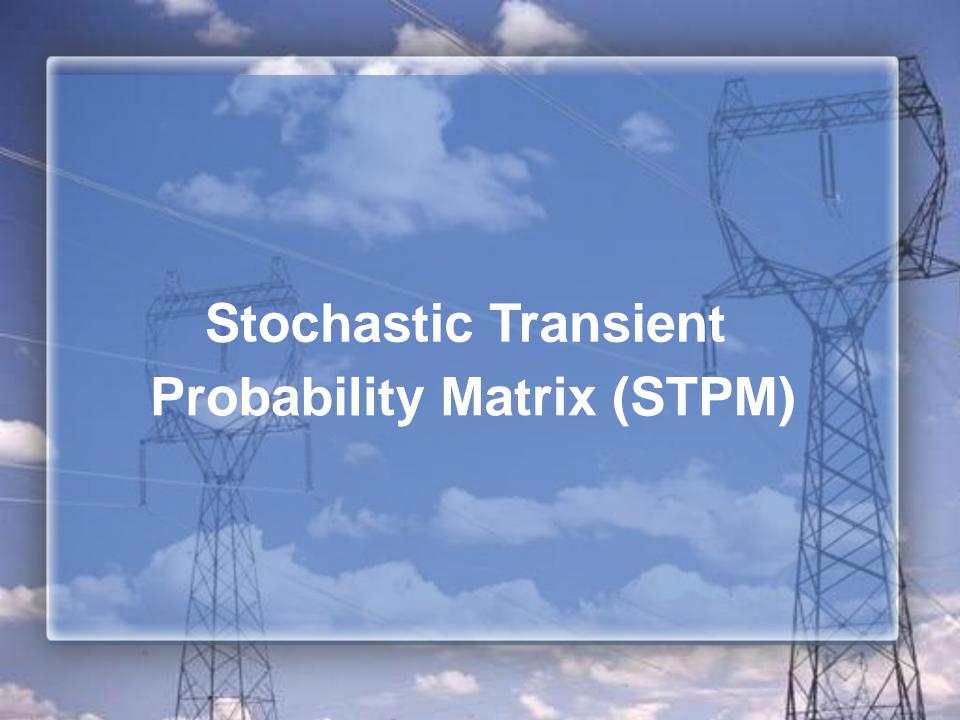
<sup>\*</sup>Failure rates for underground cable are per circuit mile.





















# □متوسط دفعات خاموشی سیستم (SAIFI)

$$SAIFI = \frac{\sum_{i=1}^{n} \lambda_i N_i}{\sum_{i=1}^{n} N_i}$$

#### System Average Interruption Frequency Index:

SAIFI = Total Number of Customer Interruptions

Total Number of Customers Served

/yr

# □متوسط زمان خاموشی سیستم (SAIDI)

$$SAIDI = \frac{\sum_{i=1}^{n} U_i N_i}{\sum_{i=1}^{n} N_i}$$

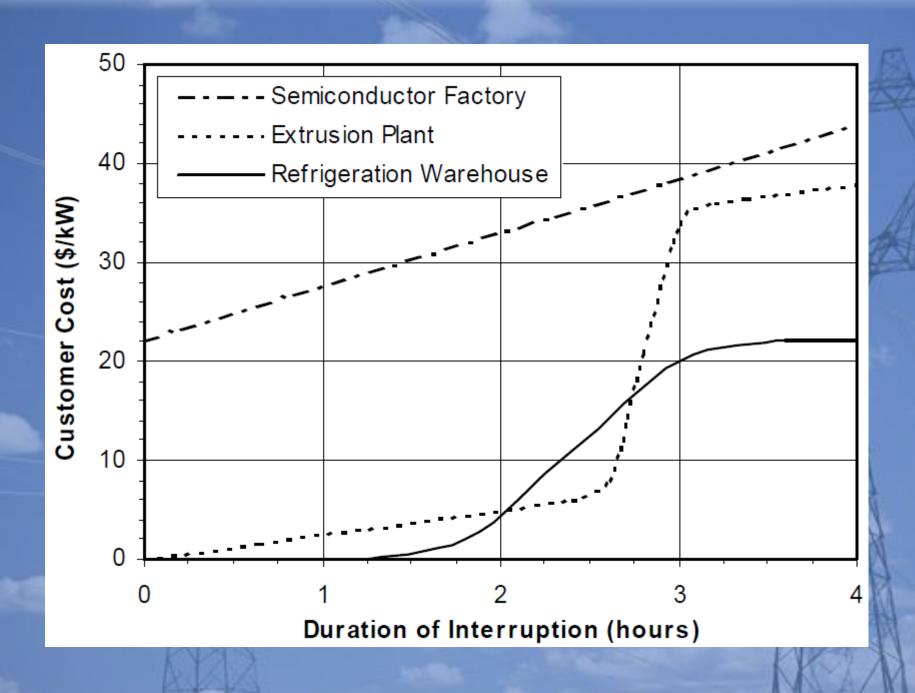
#### **System Average Interruption Duration Index:**

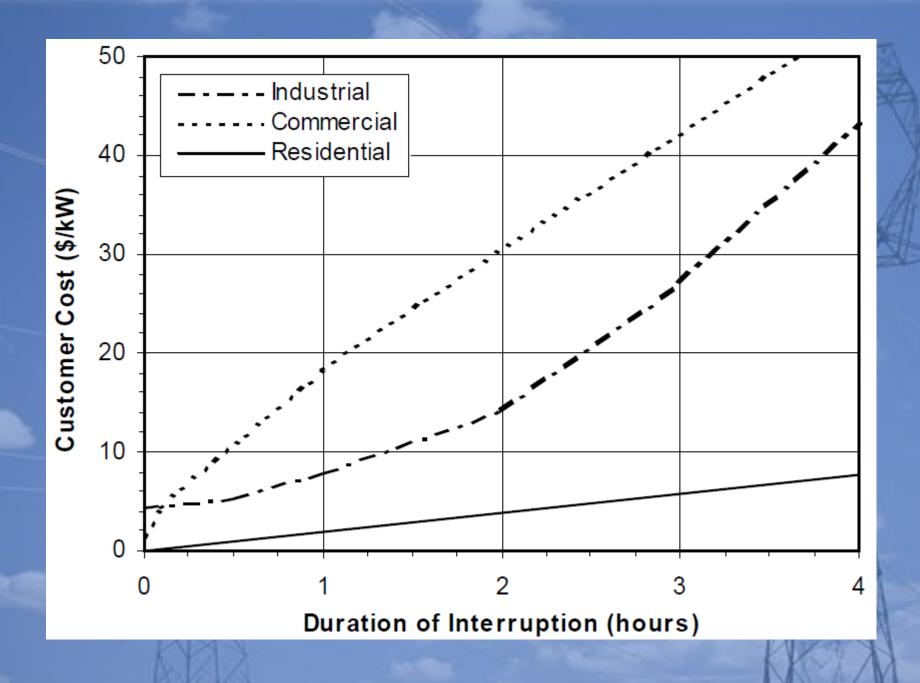
$$SAIDI = \frac{\sum Customer Interruption Durations}{Total Number of Customers Served}$$

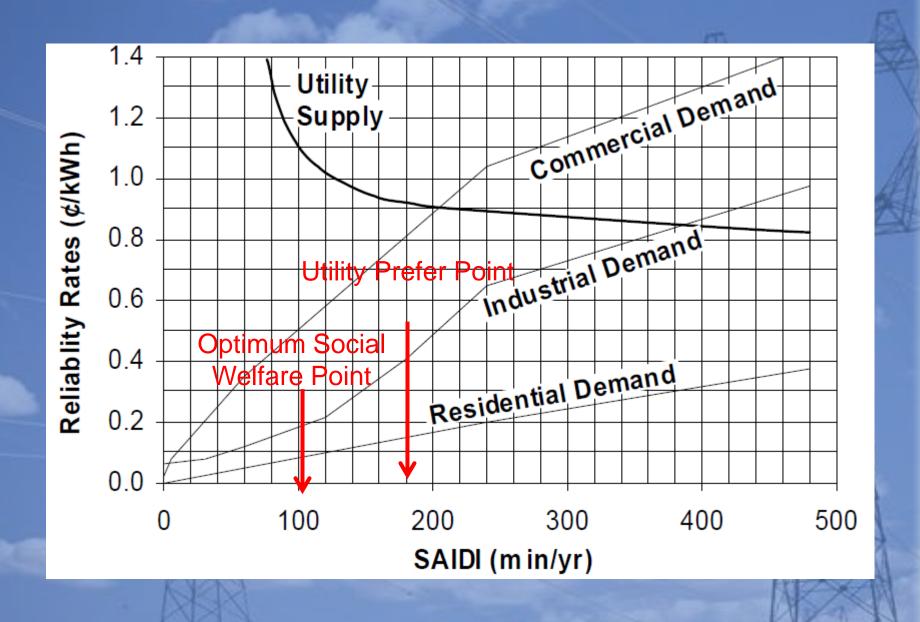
hr/yr

مفهوم وابستگی خسارت به تداوم آن و اهمیت شاخص SAIDI: برخی صنایع مثل صنعت نیمه هادی با بسروز قطعسی بلافاصسله حسداکثر خسارات را متحمل می شوند و میزان خسارت با افزایش زمان قطع بسا شیب ملایمی افزایش می یابد. برخی صنایع نیز مثل صنایع سردخانه ای و صنایع شیشسه و پلاسستیک بسه قطعیهای کوتاه مدت خیلی حساس نیستند ولی تداوم قطعسی بسه شسدن

خسارات را افزایش می دهد.







# □متوسط زمان هر خاموشی برای هر مشترک (CAIDI)

$$CAIDI = \frac{SAIDI}{SAIFI} \quad CAIDI = \frac{\sum_{i=1}^{n} U_{i} N_{i}}{\sum_{i=1}^{n} \lambda_{i} N_{i}}$$

#### **Customer Average Interruption Duration Index:**

$$CAIDI = \frac{\sum Customer\ Interruption\ Durations}{Total\ Number\ of\ Customer\ Interruptions}$$

hr

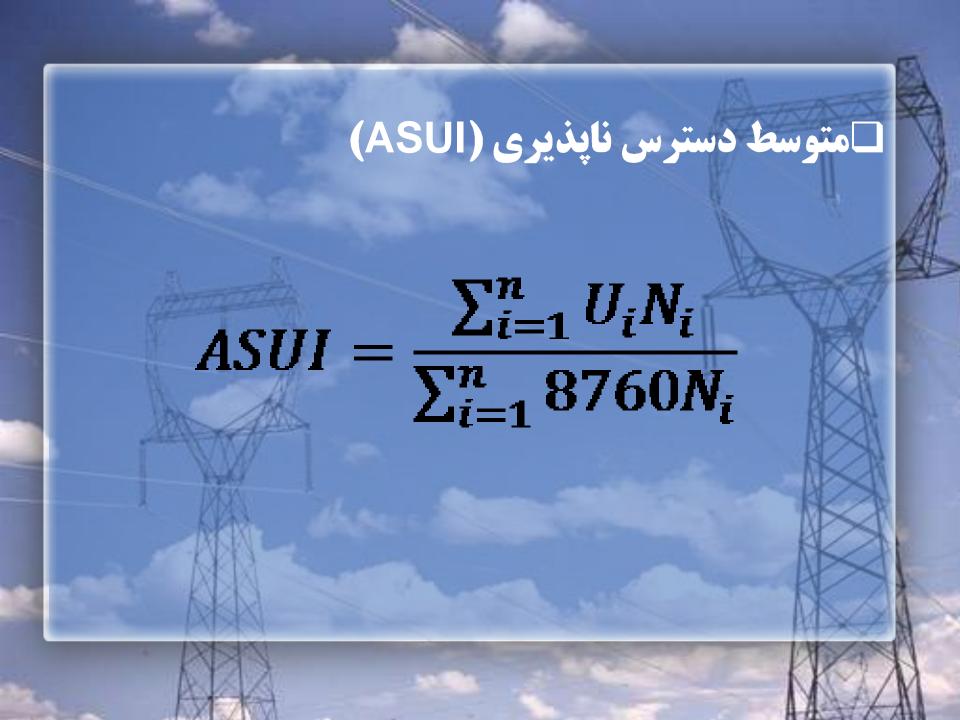
### □متوسط دسترس پذیری (ASAI)

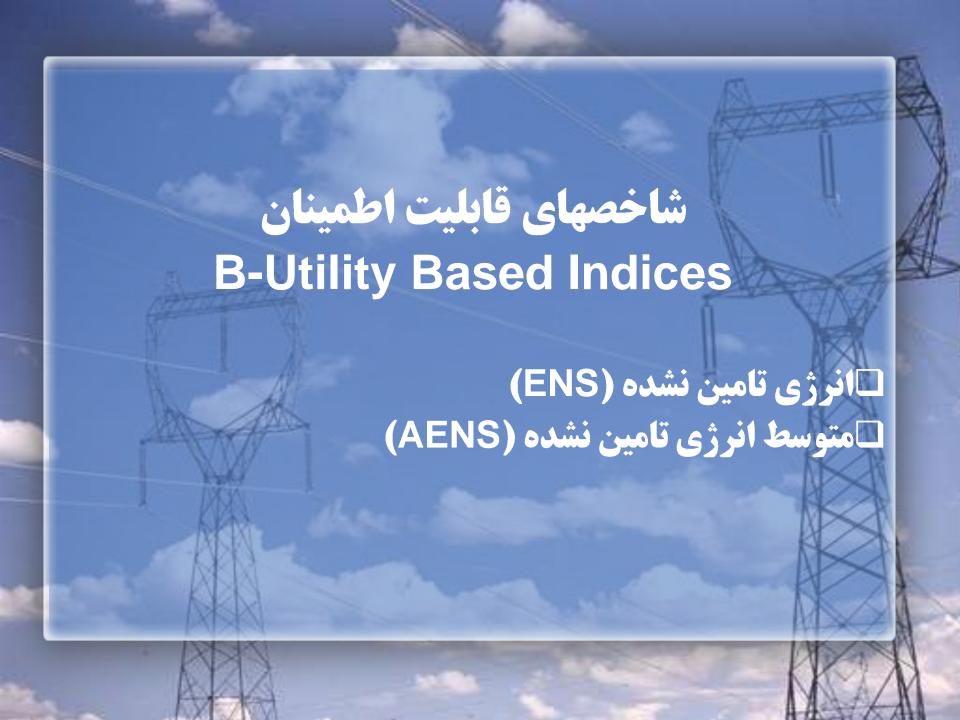
$$ASAI = \frac{\sum_{i=1}^{n} 8760N_i - \sum_{i=1}^{n} U_i N_i}{\sum_{i=1}^{n} 8760N_i}$$

#### **Average Service Availability Index:**

 $ASAI = \frac{Customer\ Hours\ Service\ Availability}{Customer\ Hours\ Service\ Demand}$ 

pu









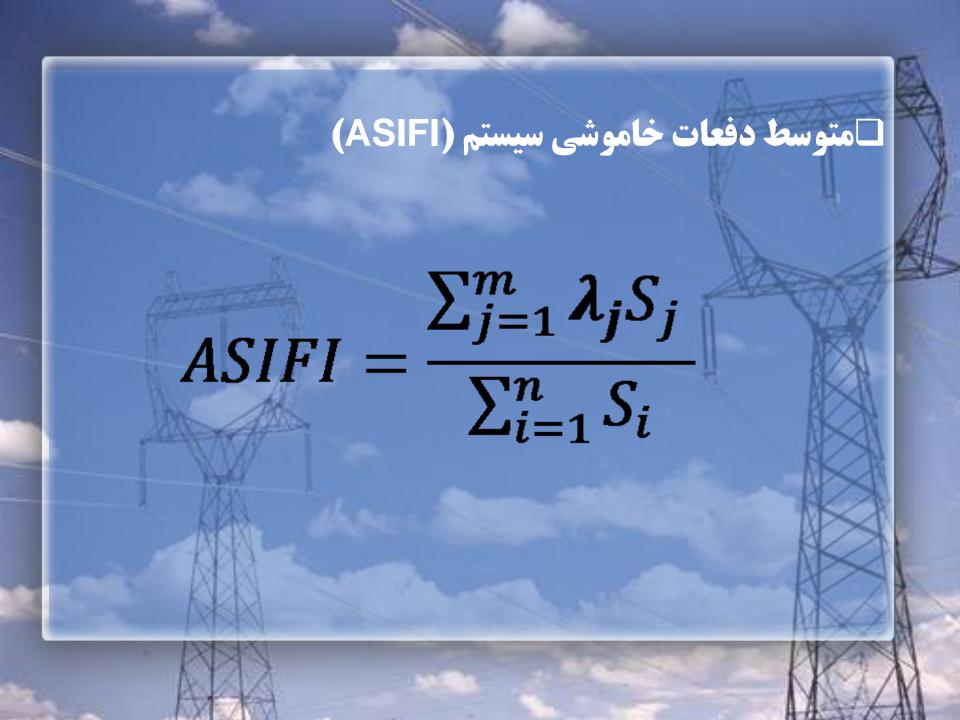


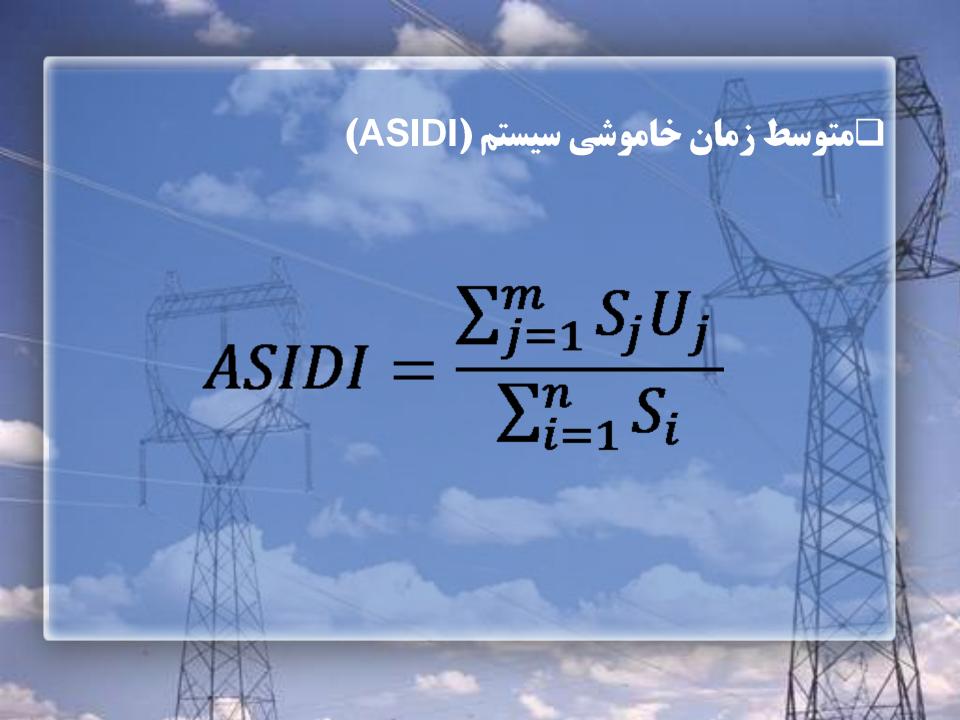


□متوسط دفعات خاموشی سیستم (ASIFI) □متوسط زمان خاموشی سیستم (ASIDI)

مشخصه: محاسبه آنها مشکل است و تنها 8% شرکتهای برق در آمریکا آنها را محاسبه می کنند.













#### **Customer Average Interruption Frequency Index:**

CAIFI = Total Number of Customer Interruptions

Customers Experiencing 1 or more Interruptions

/yr



#### **Customer Total Average Interruption Duration Index:**

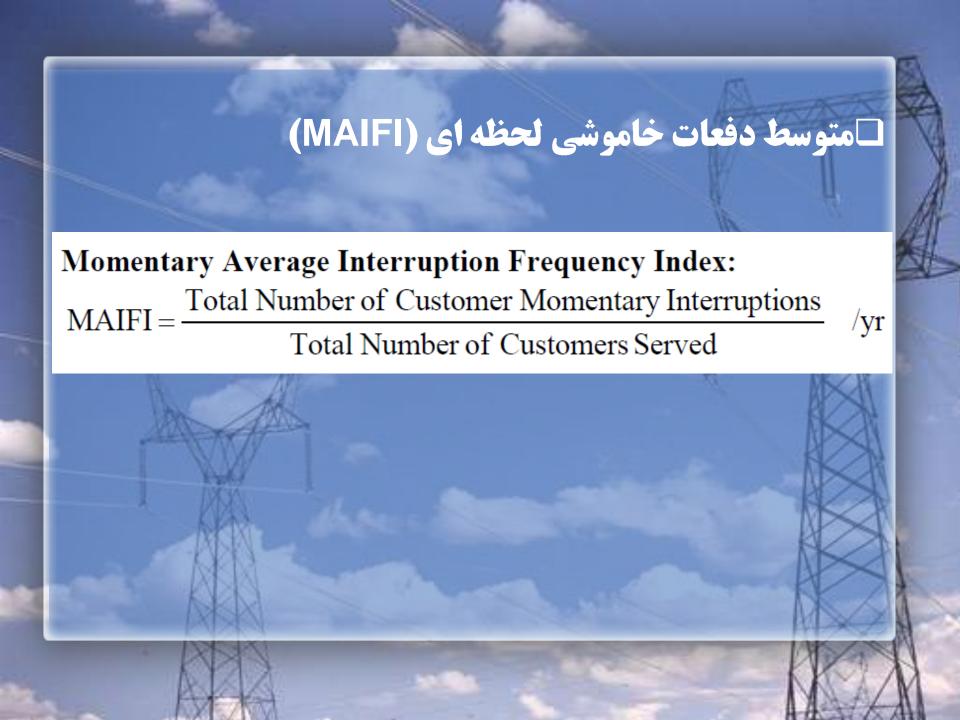
$$CTAIDI = \frac{\sum Customer\ Interruption\ Durations}{Customers\ Experiencing\ 1\ or\ more\ Interruptions}$$

hr/yr



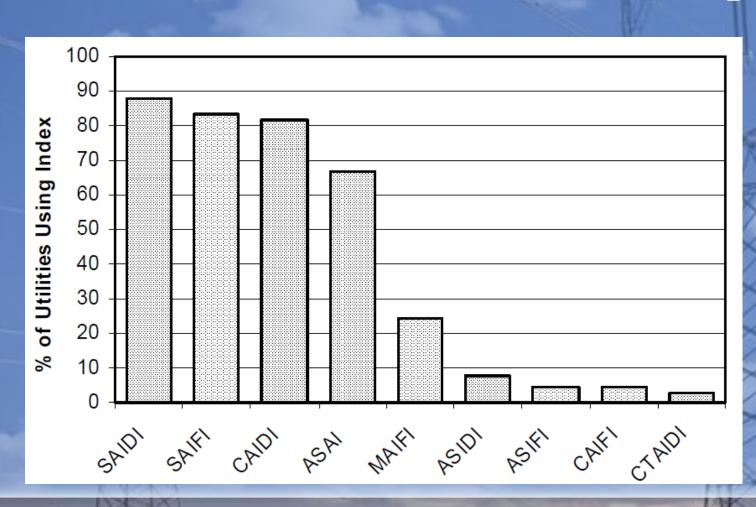
E- Momentary Interruptions Based Indices

□متوسط دفعات خاموشي لحظه اي (MAIFI)





### محبوبيت شاخصها





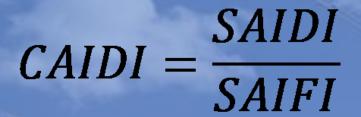


پروژه ها و هزینه کردها به سمت مناطق با چگالی جمعیت بالا سوق می یابد تا تعداد مشترکین بیشتری منتفع شوند. این امر منجر می شود که قسمتهای پرجمعیت از رشد بالاتری برخوردار شوند و سرمایه گذاریها به جای سوق یافتن به مناطق حاشیه ای به چگالتر شدن جمعیت نواحی پر جمعیت بیانجامد

## با هدف بهبود شاخصهای قابلیت اطمینان CAIDI

از دیدگاه نام، CAIDI به مفهوم متوسط زمان قطعی به ازای هر مشترک تلقی می شود ولی در واقع یک تقسیم ساده ریاضی به صورت زیر است:

$$CAIDI = \frac{SAIDI}{SAIFI}$$



لذا اگر در یک پروژه خاص SAIFI بیشتر از SAIDI کاهش یابد، به نظر می رسد که پروژه از دیدگاه CAIDI زیانبار بوده است. لذا بطور کل بیشتر شرکتها به استفاده از SAIFI و SAIDI میگیرد راغب بوده و CAIDI بسیار کمتر مورد استفاده قرار می گیرد



Table 1 shows an excerpt from one utility's customer information system (CIS) database for feeder 7075, which serves 2000 customers for a total load of 4 MW. In this example, Circuit 7075 constitutes the "system" for which the indices are calculated. More typically the "system" combines all circuits together in a region or for a whole company.



Table 1—Outage data for 1994

Date	Time	Time on	Circuit	Event code	No. of customers	Load (kVA)	Interrupt type
3/17	12:12:20	12:20:30	7075	107	200	800	Sustained
4/15	18:23:56	18:24:26	7075	256	400	1600	Momentary
5/5	00:23:10	01:34:29	7075	435	600	1800	Sustained
6/12	23:17:00	23:47:14	7075	567	25	75	Sustained
7/6	09:30:10	09:31:10	7075	567	2000	4000	Momentary
8/20	15:45:39	20:12:50	7075	832	90	500	Sustained
8/31	08:20:00	10:20:00	7075	1003	700	2100	Sustained
9/3	17:10:00	17:20:00	7075	1100	1500	3000	Sustained
10/27	10:15:00	10:55:00	7075	1356	100	200	Sustained

Table 2—Extracted customers who were interrupted

Name	Circuit no.	Date	Event code	Duration (min)	
Willis, J.	7075	3/17/94	107	8.17	
Williams, J.	7075	4/15/94	256	0.5	
Willis, J.	7075	4/15/94	256	0.5	
Wilson, D.	7075	5/5/94	435	71.3	
Willis, J.	7075	6/12/94	567	30.3	
Willis, J.	7075	8/20/94	832	267.2	
Wilson, D.	7075	8/20/94	832	267.2	
Yattaw, S.	7075	8/20/94	832	267.2	
Willis, J.	7075	8/31/94	1003	120	
Willis, J.	7075	9/3/94	10	10	
Willis, J.	7075	10/27/94	1356	40	



Table 3—Interrupting device operations

Device	Date	Time	No. of operations	No. of operations to lockout
Brk 7075	4/15	18:23:56	2	3
Recl 7075	7/6	09:30:10	3	4
Brk 7075	8/2	2:29:02	1	3
Brk 7075	8/2	2:30:50	2	3
Recl 7075A	8/2	3:25:40	2	4
Recl 7075	8/25	08:00:00	2	4
Brk 7075	9/2	04:06:53	2	3
Recl 7075	9/5	11:53:22	3	4
Brk 7075	9/8	5:25:10	1	3
Recl 7075	10/2	7:15:19	1	4
Recl 7075	11/12	00:00:05	1	4

To better illustrate the concepts of momentary interruption, and sustained interruption, and the associated indices, consider the following figure.

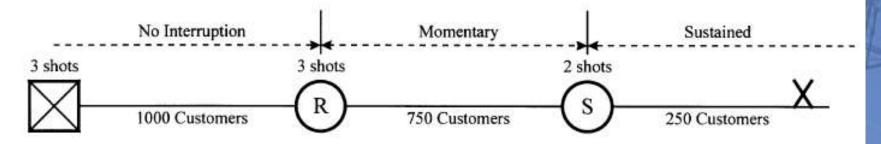


Figure 1—Sample system 2

For this scenario, 750 customers would experience a momentary interruption and 250 customers would experience a sustained interruption. Calculations for SAIFI, MAIFI, and MAIFI<sub>E</sub> are shown below.



### مقایسه شاخصهای قابلیت اطمینان در سال ۱۹۸۷

**Table 2.6.** Distribution reliability indices reported from utilities around the world (1987 data). The reliability of rural systems tends to get better as population density increases. The notable exception is Italy, which has poor reliability in both its urban and rural areas.

	SAIFI	SAIDI	CAIDI	ASAI	Density (people/mi <sup>2</sup> )
	(/yr)	(min/yr)	(min)	(pu)	
Urban Systems	62 5355—33 A	M SAME THROUGH		M SOUTH	R 4R & 0
Finland	0.8	33	41	0.99994	
Sweden	0.5	30	60	0.99994	
Denmark	0.3	7	20	0.99999	
Italy	2.5	120	48	0.99977	
Netherlands	0.3	15	58	0.99997	
Rural Systems					
Finland	5.0	390	78	0.99926	38.3
Sweden	1.5	180	120	0.99966	51.2
Denmark	1.2	54	45	0.99990	313.7
Italy	5.0	300	60	0.99943	496.9
Netherlands	0.4	34	79	0.99994	975.3
Overall					
Norway	2.0	300	150	0.99943	34.6
United States	1.3	120	90	0.99940	73.2
United Kingdom	0.7	67	92	0.99987	653.4
Netherlands	0.4	27	73	0.99995	975.3